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Laminar, transitional and turbulent friction factors for gas flows in smooth and rough microtubes

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ABSTRACT

Theoretical and experimental works on microscale transport phenomena have been carried out in the past decade in the attempt to analyze possible new effects and to assess the influence of downscaling on the classical correlations which are used in macro-scale heat and fluid flow, following the need to supply engineers with reliable tools to be used in the design of micro-scale devices. These results were sometimes in mutual contrast, as is the case for the determination of the friction factor, which has been found to be lower, higher or comparable to that for macroscopic channels, depending on the researchers. In this work the compressible flow of nitrogen inside circular microchannels from 26 μm to 508 μm in diameter and with different surface roughness is investigated for the whole range of flow conditions: laminar, transitional and turbulent. Over 5000 experimental data have been collected and analysed. The data confirmed that in the laminar regime the agreement with the conventional theory is very good in terms of friction factors both for rough and smooth microtubes. For the smaller microchannels (<100 μm) when Re is greater than 1300 the friction factor tends to deviate from the Poiseuille law because the flow acceleration due to compressibility effects gains in importance. The transitional regime was found to start no earlier than at values of the Reynolds number around 1800. Both smooth and sudden changes in the flow regime have been found, as reported for conventional tubes. Fully developed turbulent flow was attained with both smooth and rough tubes, and the results for smooth tubes seem to confirm Blasius' relation, while for rough tubes the Colebrook–White correlation is found to be only partially in agreement with the experimental friction factors. In the turbulent regime the dependence of the friction factor on the Reynolds number is less pronounced for microtubes than the prediction of the Colebrook–White correlation and the friction factor depends only on the microtube "relative roughness".

1. Introduction

In the last years a large amount of experimental analyses have been addressed to the study of the frictional characteristics of liquid and gas flows in microchannels, with a chronologically decreasing discrepancy between experiments and theoretical results. This fact can be explained with the improvement of the techniques for microfabrication with a consequent more accurate control of surface roughness and channel geometry. However, it is the authors' opinion that some systemic studies are still needed, both to give the whole subject a coherent treatment and to further investigate some points, such as the laminar-to-turbulent transition and the reliability of conventional correlations in the developing and fully developed turbulent regime, since these are still controversial.

Recent reviews of the experimental results related to the laminar-to-turbulent transition published in the last years are due to Morini [1,2] and Hetsroni et al. [3]. These studies indicate that the transition from the laminar to the turbulent flow in micro-scale passages could take place at critical Reynolds numbers ranging from 70 (Peng and Peterson [4]) up to 10,000 (Stanley et al. [5]). This large spread of the critical Reynolds values has stimulated this work with the aim to experimentally add a contribution in terms of data and comments to the region and form of the onset of such a transition. The first work in which the friction factor through rectangular and trapezoidal glass and silicon microchannels was measured for different gases (N_2, H_2, Ar) is due to Wu and Little [6]; the measured values of the friction factor were larger (10%–30%) than those predicted by the conventional theory. The glass channels they employed had a rectangular cross section with two rounded corners and with a high "relative roughness" ($\epsilon/D = 0.2$–0.3) non uniformly distributed along the wetted perimeter. The silicon microchannels were chemically etched on a ⟨100⟩ oriented wafer; in this case the channels were trapezoidal and could be considered smooth.

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Nomenclature

A	cross-sectional area, m^2
C^*	Poiseuille numbers ratio, Eq. (3)
D	inner diameter, m
f	Darcy–Weisbach friction factor
FS	full scale
L	microtube length, m
$\dot{m}$	mass flow rate, kg s^{-1}
p	pressure, Pa
Re	Reynolds number

Greek symbols

Δ	difference
ε	absolute roughness, m
μ	dynamic viscosity, Pa s
ρ	gas density, kg m^{-3}

Subscripts

c	critical
h	hydraulic
i	inlet
m	measured
n	net
no	nominal
o	outlet

Comparing the results of Wu and Little [6] for glass and silicon channels, the role of the "relative roughness" on transition was evidenced. Wu and Little concluded that transition occurred at Reynolds numbers ranging from 1000 to 3000. Acosta et al. [7] presented an analysis of the friction factors for isothermal gas flows in rectangular microchannels; the investigated channels had a very small aspect ratio. The tests evidenced trends which were very similar to those for conventional laminar-to-turbulent transition; the critical Reynolds number was about 2770, as quoted by Obot [8]. Choi et al. [9] measured the friction factor for the fully developed laminar flow of nitrogen through silica micropipes having a diameter of 3, 7, 10, 53, 81 μm with Reynolds numbers ranging between 30 and 20,000. Their data suggested that the critical Reynolds number decreased with the hydraulic diameter; in particular, the transition occurred at Reynolds equal to 2000 for a circular tube with a hydraulic diameter of 53 μm and at 500 for a hydraulic diameter of 9.7 μm. Yu et al. [10] tested nitrogen and water flows through silica microtubes having a diameter of 19, 52 and 102 μm for Reynolds numbers ranging between 250 and 20,000. They found that transition occurred for Reynolds numbers between 1700 and 6000, in line with the conventional theory for continuum flows. Stanley et al. [5] carried out experiments on liquid and gas flow in rectangular microchannels having a hydraulic diameter from 56 μm to 260 μm. Transition was reported for nitrogen flows at values of the Reynolds number between 1500 and 2000 when the hydraulic diameter was larger than 150 μm. Li et al. [11] deduced from their experimental results on the friction factors for circular microtubes that the transition in microtubes occurs at a Reynolds number equal to 2300. In a further work [12] the same authors concluded that the transition from laminar to turbulent occurred at Reynolds numbers between 1700 and 2000. Yang et al. [13] experienced that transition occurred when the Reynolds numbers varied from 1200 to 3800 for air, water and R-134a through microtubes. The range of critical Reynolds numbers increased with the decrease in tube diameter. This dimensional dependence was more marked for water flow than for air flow. Li et al. [14] observed that for smooth microtubes with a hydraulic diameter ranging between 79.9 and 166.3 μm transition occurred at Reynolds numbers equal to 2000–2300; this fact underlined that the conventional theory for incompressible laminar flow still worked for microtubes with diameters larger than 80 μm. They also observed that for rough stainless steel microtubes ($D_h = 136.5–179.8$ μm) having a "relative roughness" equal to 5% the flow transition occurred at lower Reynolds number ($\approx 1700–1900$). By analyzing their experimental data, the authors remarked that the conclusion of an early transition for flows in rough microtubes could not be drawn with certainty. Faghri and Turner [15] investigated the effect of relative surface roughness in microchannels etched in ⟨100⟩ (leading to trapezoidal cross sections) and ⟨101⟩ (leading to rectangular cross sections) silicon wafers. They fabricated various microchannels having different surface roughness and tested them with nitrogen and helium. They found that the effect of "relative roughness" on the friction factor was negligible for values lower than 6% and their results were in a good agreement with the conventional theory. Tang and He [16] measured friction factors for nitrogen flows in fused silica microtubes and square microchannels with diameters ranging from 50 μm to 201 μm. They concluded that the theoretical prediction for conventional tubes are in good agreement with the measured friction factor but for smaller microtubes the effects related to compressibility, roughness and rarefaction could not be neglected. The laminar-to-turbulent transition was experimentally evidenced for critical Reynolds numbers ranging between 1900 and 2500. Kohl et al. [17] studied the flow of water and air in rectangular silicon microchannels with a hydraulic diameter varying between 24.9 and 99.8 μm and performed internal pressure measurements. They could predict their data quite well using the classical correlations for laminar flows. None of their experimental results indicated an early transition to turbulence. However, they remarked that when the compressibility effects became significant the L/D (length-to-diameter) ratio could influence the critical Reynolds number. Their values of the critical Reynolds number obtained for air ranged between 2300 and 6000 as a function of the L/D ratio. The authors explained this behavior by invoking the large accelerations in the microtube, as theoretically predicted by Kurokawa and Morikawa [18] and Schwartz [19].

From the above analysis one can conclude that the flow behavior in the transitional region is still an open issue for microchannels: this flow regime is more frequent for gases at the microscale owing to the flow velocities and corresponding Reynolds numbers involved. Moreover, although it seems that there is a widespread agreement that the friction factor for the laminar regime is correctly predicted by traditional correlations, provided all effects are accounted for (and this is not always the case, even in very recent works like Dutkowski [20], where e.g. inlet and outlet losses and compressibility are not considered), the influence of surface finish, which can involve very high values of roughness as this term is understood in mechanical manufacturing, is still an interesting issue, which has been granted special attention by Kandlikar and co-workers [21,22], and data on rough tubes add elements to the discussion in this area. When it comes to turbulent flow, the matter becomes more involved as there is considerable misunderstanding and misuse in the notion of "relative roughness", which is normally taken as the ratio of the surface roughness as measured by a profilometer or equivalent instrument to the characteristic dimension of the tube or channel. This is unfortunately not the same as the parameter introduced by Colebrook in his paper on transitional friction factor [23], which makes comparison with this correlation very tricky. This work on the compressible flow of gaseous nitrogen within circular microchannels with both smooth and rough walls is meant to be a contribution to the subject.

2. The experimental apparatus

The sketch of the apparatus is shown in Fig. 1. Nitrogen is stored in two pressurised tanks (20 bar and 10 bar) (1*a*–1*b*). Two 7 μm particle filters (2, *Hamlet*) are located along each pipeline before the flow meters to prevent possible impurities from clogging the microchannels. Ball valves (3) are used in order to select the feeder and the proper flow sensor, as the facility is equipped with three *Bronkhorst EL-Flow E7000* operating in the 0–5000 nml/min (4*a*) in the 0–500 nml/min (4*b*) and in the 0–50 nml/min (4*c*) range respectively. The flow sensors can also dictate the mass flow rate through a computer-steered valve: this allows an indirect regulation of the pressure at the inlet of the microtube: this is an important point, as all classical studies on friction factor and onset of turbulence were carried out at constant pressure difference between reservoir (inlet) and outlet. The gas then enters the test section (5), before which a pressure tap is located. The test section allows tubes of varying length (1–100 cm) and variable inner diameter to be tested, providing they have the same external diameter, namely 1/16″. After exiting the microtube, the gas vents to the atmosphere. The total pressure drop between the inlet and the outlet of the microtube is measured by a differential pressure transducer (6, *Validyne DP15*) with an interchangeable sensing element, to allow accurate measurements over the whole range of pressures encountered. To measure the temperature at the entrance and exit of the microtube, two K-type, calibrated thermocouples are used (7): through these values and that of the pressure, the densities of the gas at the channel inlet and outlet can be calculated. During the experimental runs the microtube to be tested is first mounted on the rig and the circuit is checked for leakages; measurements are then carried out by imposing a certain inlet Reynolds number (which can be related to the mass flow rate, the parameter actually controlled) and recording for every measurement pressure drop, time, inlet and outlet temperature and flow rate by means of a data acquisition system and a PC.

Seven different commercial microtubes (*Upchurch Inc.*) were tested in the experiments: three fused silica microtubes (FS), two fused silica microtubes coated in Peek (PS) and two stainless steel microtubes. The main geometrical characteristics of the tested microtubes are stated in Table 1 (nominal diameter (D_{no}), length (L), actual diameter (D), length-to-diameter ratio (L/D), range of Reynolds numbers).

The inner diameter of the frontal cross-section of each microtube has been verified by means of a SEM microscope (*JEOL JSM 5200*).

The pictures confirm that the fused silica microtubes can be considered completely smooth for all practical purposes; on the contrary, stainless steel commercial microtubes are characterized by very irregular walls. The SEM images of each microtube have been used in order to verify the inner diameter and the average height of the roughness by superimposing a best-fitting circumference over the corresponding contours.

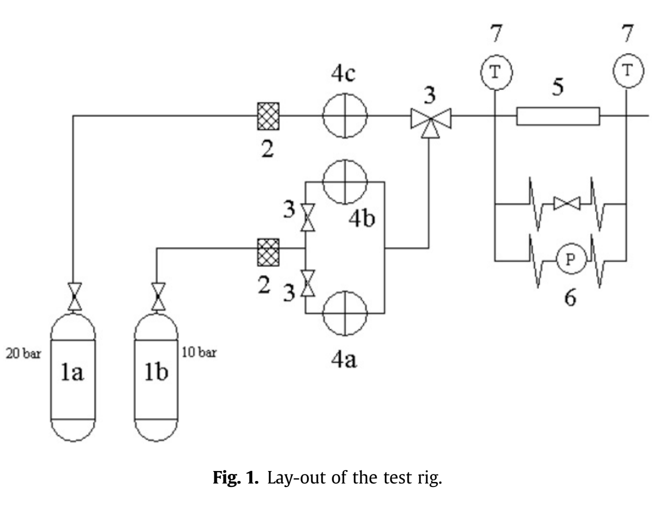

Fig. 1. Lay-out of the test rig.

Table 1
Geometrical characteristics of the tested microtubes.

Name	D_{no} (μm)	L (cm)	D (μm)	L/D	Reynolds range
FS1	20	5.05	29.9	1689	70–1050
FS2	50	4.90	51.4	953	50–2760
FS3	100	10.00	100	1000	20–4050
PS1	25	5.00	26.1	1916	90–750
PS2	50	5.00	52.2	958	30–3034
SS1	250	20.00	275	727	400–9500
SS2	500	30.00	508	591	1800–14,500

It has been observed that for the commercial microtubes the nominal value of the inner diameter (D_{no}) declared by the manufacturer can be very different from the actual value (D). The values of the inner diameter determined by analyzing the SEM images of the microtubes are shown in Table 1.

It is interesting to note that the difference (in percent) between the measured inner diameter and the nominal one varies from 0.0 to 49.9%, the latter being the case for tube FS1. The average height of the roughness (ϵ) of the stainless steel microtubes is of the order of 3–4 μm.

3. Data reduction and uncertainty analysis

For channels which are extremely long compared to their hydraulic diameter, there is sufficient area for heat transfer so that the flow is with good approximation isothermal; since the microtubes tested in this work are characterized by large values of the L/D ratio (>600) the flow can be modeled as such. Shapiro [24] presents the following equation to calculate the friction factor for an isothermal compressible flow in a constant-area duct

$$f = \frac{D}{L}\left[\left(\frac{1-\left(1-\frac{\Delta p_n}{p_i}\right)^2}{\left(\frac{\dot{m}\sqrt{RT}}{Ap_i}\right)^2}\right) - 2\ln\left(\frac{1}{1-\frac{\Delta p_n}{p_i}}\right)\right] \tag{1}$$

The value of the net pressure drop (Δp_n) is deduced from the measured differential pressure as indicated in the data reduction procedure described by the authors in previous works [25,26].

For viscous, compressible flows the Reynolds number could change between inlet and outlet of the tube. In this work the friction factor is referred to the value of the Reynolds number defined as follows:

$$Re = \frac{\dot{m}D}{A\mu} \tag{2}$$

For isothermal flows the Reynolds number defined by Eq. (2) is constant along the microtube. To assess the accuracy of measurements presented in this work, the uncertainty associated to each measurement device is reported in Table 2.

The uncertainty in measuring f and Re can be estimated by applying the theory on the propagation of errors (Moffat, [27]) to the uncertainty associated with the measurement of the single quantities p, D, L, $\dot{m}$ and so on. The procedure adopted in this work has been detailed by the authors in [28]. It has been demonstrated that the measurement of the diameter is the most critical one for the overall measurement uncertainty. In the present case, the uncertainty on the inner diameter evaluated using a SEM is around ±2%; the uncertainty on the length is of the order of ±0.3%.

Table 2
Characteristics and uncertainties of the measurement instruments.

Label	Instrument	Range	Uncertainty
(4a)	Flow meter	0–5000 (nml/min)	±0.6% FS
(4b)	Flow meter	0–500 (nml/min)	±0.5% FS
(4c)	Flow meter	0–50 (nml/min)	±0.45% FS
(6)	Differential pressure transducer	0–35 (kPa) 0–86 (kPa) 0–220 (kPa) 0–860 (kPa) 0–1440 (kPa)	±0.5% FS
(7)	Thermocouple	0–200 (°C)	±0.25% FS

4. Results and discussion

The experimental campaign has been conducted to ascertain whether the friction factor in the laminar and turbulent regimes obeys the laws validated for conventional smooth and rough tubes. To this aim, the Moody chart has been used to plot the experimental data. In the diagram the line representing the Poiseuille law for circular tubes ($f = 64/\text{Re}$) for $\text{Re} < 2000$ is drawn together with the Blasius law ($f = 0.316/\text{Re}^{0.25}$) for smooth tubes in the turbulent regime. The former curve is represented by a continuous line, while the latter is dashed.

4.1. Laminar regime

Fig. 2 shows the friction factors which were obtained for the FS1 microchannel considering an inner nominal diameter as declared by the manufacturer of 20 μm. If the nominal diameter is used to calculate the friction factor for nitrogen in laminar flow the experimental data are manifestly overpredicted by the Poiseuille law. On the contrary, using the inner diameter calculated through SEM imaging ($D = 29.9$ μm) the experimental friction factors turn out to be in excellent agreement with the classical correlation. This fact evidences the importance of verifying, especially for very small commercial microtubes, the inner dimensions. In the following figures the friction factor is always calculated using the measured value of the inner diameter, which is reported for each tube in Table 1. Also corrections for compressibility and inlet and outlet pressure losses must be taken into account when extracting the friction factor for the raw data: disregarding these corrections has been, and still is, with a somewhat rarer occurrence, the cause for discrepancies between theoretical predictions and experimental calculations.

In Fig. 3 the friction factors obtained for the tested rough and smooth microtubes in the laminar regime are drawn on the Moody chart and compared with the Poiseuille law. It is evident how in the log–log plot the experimental data are in very good agreement with theoretical predictions. Yet, this kind of representation pinches the difference between the Poiseuille law and the experimental data. For this reason, the recorded data have been used in order to calculate the ratio

$$C^* = \frac{f\text{Re}}{64} \tag{3}$$

between the measured Poiseuille number, defined as product of the Darcy–Weisbach friction factor times the Reynolds number, and the theoretical value for an incompressible laminar flow through a circular tube ($f\text{Re} = 64$).

In Fig. 4, C^* is plotted as a function of the Reynolds number. Only the error bars of the experimental data referring to the FS2 microtube are shown to improve the readability. For the FS2 microtube the uncertainty on the Poiseuille number varies between ±9% ($\text{Re} > 500$) and ±59% ($\text{Re} = 30$). The larger values of the uncertainty for low Reynolds are due to the increase of the uncertainty on the flow rate when operating conditions are far from the full scale value of the flow meter used in the test (50 nml/min, the smallest available). This fact justifies the generalized scattering of the points for Reynolds numbers lower than 90. In the range $90 < \text{Re} < 1300$ the agreement between the experimental data and the theoretical predictions is within experimental uncertainty. On the contrary, for Re greater than 1300–1500, especially for smaller microtubes, the value of the experimental data exhibits an increase of the Poiseuille number with the increase of the Reynolds number. This trend is not related to the onset of an earlier laminar-to-turbulent transition but it is due to compressibility. When the Reynolds number increases the average fluid velocity increases together with the Mach number. The flow is accelerated by the decrease in the fluid density in the flow direction and this acceleration increases the velocity gradient near the walls and hence the friction factor. As demonstrated theoretically by Asako et al. [29] and Li et al. [30] the Poiseuille number in the laminar regime tends to increase with the Mach number when the average Mach number

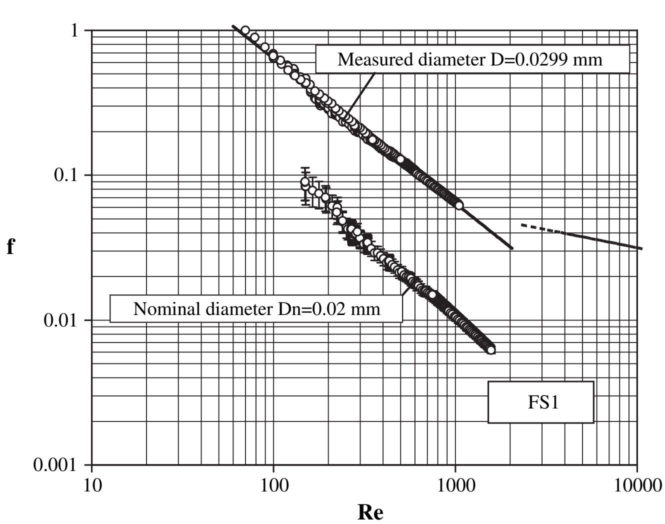

Fig. 2. Friction factors of the FS1 microtube calculated using the nominal and the measured inner diameter.

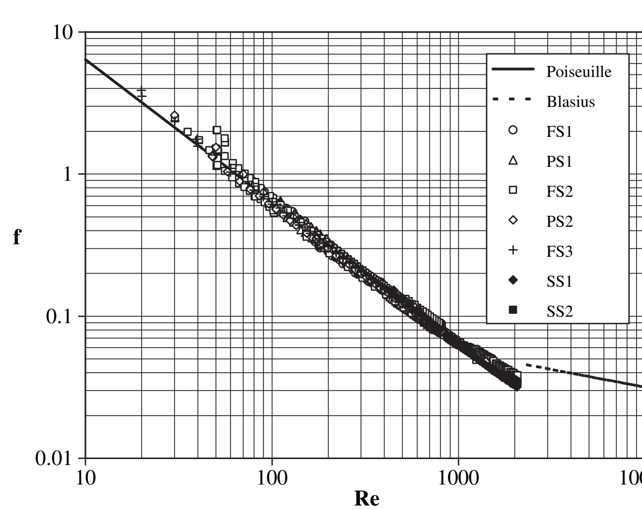

Fig. 3. Comparison between the experimental data and Poiseuille law in the Moody diagram.

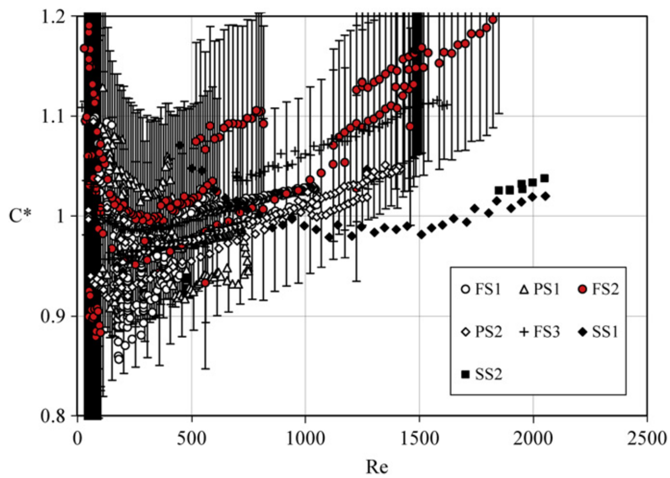

Fig. 4. The coefficient C^{*} defined by Eq. (3) as a function of the Reynolds number.

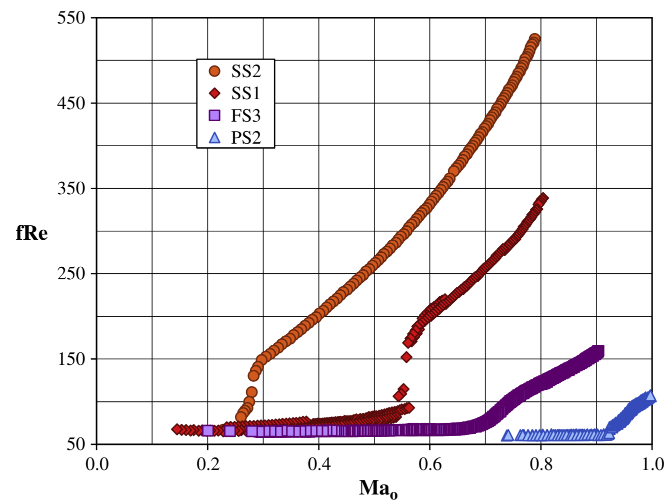

Fig. 5. Poiseuille number as a function of the outlet Mach number.

between the inlet and the outlet becomes larger than 0.3. Comparison between the above correlations and the experimental data has already been carried out in [26], and the data show a sharp change in the slope when transition to turbulence sets in. For the microtubes tested the average Mach number varied between 0.1 and 0.25, but with exit values often in the range 0.6–0.98, especially for the tubes of smaller diameter, and the compressibility effects become felt, as confirmed by the data plotted in Fig. 4.

Fig. 4 shows how for the rough microtubes (full symbols) the increase of the Poiseuille number for increasing Reynolds numbers is less evident than for smooth microtubes. It is important to observe that the tested rough microtubes (SS) have a larger inner diameter than the FS and PS microtubes and hence, for a fixed Reynolds number, in the rough microtubes the Mach number is lower. In other words, the compressibility effects are less important in the laminar regime ($0 < Re < 2000$) for microtubes with an inner diameter greater than 200 μm.

The plots in Fig. 4 evidence that the agreement with the Poiseuille law is good even for rough microtubes; this fact leads to the conclusion that the friction factor in the laminar regime can be considered independent on the surface finish, at least for the cases of the tubes tested. This confirms the numerical results obtained, among others, by Gamrat et al. [31] and Croce et al. [32].

The value of the Poiseuille number as a function of the outlet Mach number is shown in Fig. 5 for the four microtubes where the experimental conditions allowed turbulent flow to develop. It is again clear how the smaller diameter tubes have larger exit Mach numbers, which indicates the presence of compressibility effects. From the same figure the two kind of transition to turbulence can be seen: a smooth change in the case of FS3 and PS2, and a sharp one in the case of SS1 and SS2.

4.2. Laminar-to-turbulent transition

In Fig. 6 the experimental data obtained in the range of the Reynolds number between 1500 and 6500 (transitional region) are plotted in a Moody chart and compared with the Poiseuille law (smooth tubes, laminar regime) and the Blasius law (smooth tubes, turbulent regime). The design of the test rig enabled the observation of the laminar-to-turbulent transition only in the FS2, PS2, FS3, SS1 and SS2 microtubes.

As evidenced by Kandlikar in [22] and by Asako et al. in [29], two different kinds of laminar-to-turbulence transition can be experienced in microtubes:

- *Smooth laminar-to-turbulent transition (ST)*, in which the transition gradually occurs.
- *Abrupt laminar-to-turbulent transition (AT)*, in which the transition suddenly occurs.

Considering the first kind of transition one can associate the end of the laminar zone and the onset of the fully developed turbulent regime respectively to the minimum and the maximum of the friction factor when the flow is subjected to the gradual transition between the regimes. On the contrary, when the laminar-to-turbulent transition occurs abruptly, the transition region appears in the Moody chart to be very limited in range. This difference is well evident in Fig. 5.

The experimental data plotted in Fig. 6 show how the laminar-to-turbulent transition takes place at Reynolds numbers larger than 2100 for all the tested microtubes. This result is in good agreement with the typical observations for conventional tubes and with previous data obtained by the authors.

As evidenced by the plots of Fig. 6, both the smooth (ST) and the abrupt (AT) transition have been observed in the tests. In particular, the smooth microtubes (FS2, PS2 and FS3) exhibited smooth transition from the laminar to the turbulent regime in which the

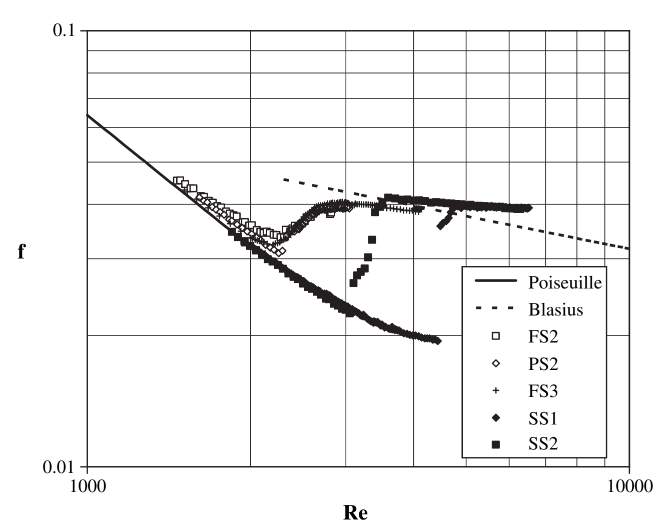

Fig. 6. Friction factor as a function of the Reynolds number in the transition region.

friction factor gradually increases during the change. The critical Reynolds number, intended as the value for which departure from the laminar correlation starts, for the smooth microtubes is equal to 2270 (FS2), 2290 (PS2) and 2160 (FS3): the behavior of these three microtubes during transition is very similar.

The stainless steel microtubes evidenced an abrupt transition at larger critical Reynolds numbers. The critical Reynolds number is equal to 4370 for the SS1 microtube and 2950 for the SS2 microtube. This result seems to be in contradiction with the observation that the transition is generally anticipated in rough microtubes because the surface roughness facilitates the detachment of the boundary layer near the walls. It is however to be mentioned that in this case the roughness of the stainless microtubes tested was limited and that the authors have experienced the abrupt transition even in smooth microchannels in other experimental works [26].

In Fig. 7 the critical Reynolds numbers obtained in previous works by the authors for smooth and rough microtubes are plotted together with the results presented in this paper as a function of the *L*/*D* ratio.

Before further commenting on Fig. 7, the criteria for the determination of critical Reynolds number in microchannels as suggested by Kandlikar [22] should be recalled. For the two rough tubes tested in this study, the average roughness amounts to 2.8 μm for SS1 and 4 μm for SS2, which would correspond to critical Reynolds numbers of 2105 and 2150 respectively, whereas what is found experimentally is 4370 and 2950. What is more important, however, is that the correlation given in [22] sets 2300 as the upper bound for Re_c in rough tubes (with values going as low as 800 for a "relative roughness" of 0.08), whereas the data plotted clearly demonstrate how transition does occur also at Reynolds numbers considerably above 2300. This, in the Authors' opinion, would suggest that more work on the subject is needed, and probably getting values of "relative roughness" from surface measurements – besides being practically impossible – is not the best choice, as shall be discussed further when dealing with the turbulent regime.

All the rough microtubes are characterized by ratios between roughness average height and diameter less than 0.05, with an inner diameter ranging between 100 and 1000 μm. The critical Reynolds numbers fall between 1506 and 4370. In Fig. 7a, where the data are divided between rough and smooth microtubes, in spite of considerable scatter, it is reasonable to state that early turbulent transition can only be achieved in rough pipes, which is coherent with the common notion of roughness anticipating transition. Yet, it is to be remarked that this never occurs before Re = 1500, which is still far removed from the results of Peng and Peterson [4]. Moreover, even in the case of rough pipes transition can be postponed to higher Reynolds numbers, indeed the largest values of Re_c are associated with rough pipes at low values of *L*/*D*. Although there is no apparent explanation for this, it may be a hint that it is not only the length-to-diameter ratio that plays a role, but

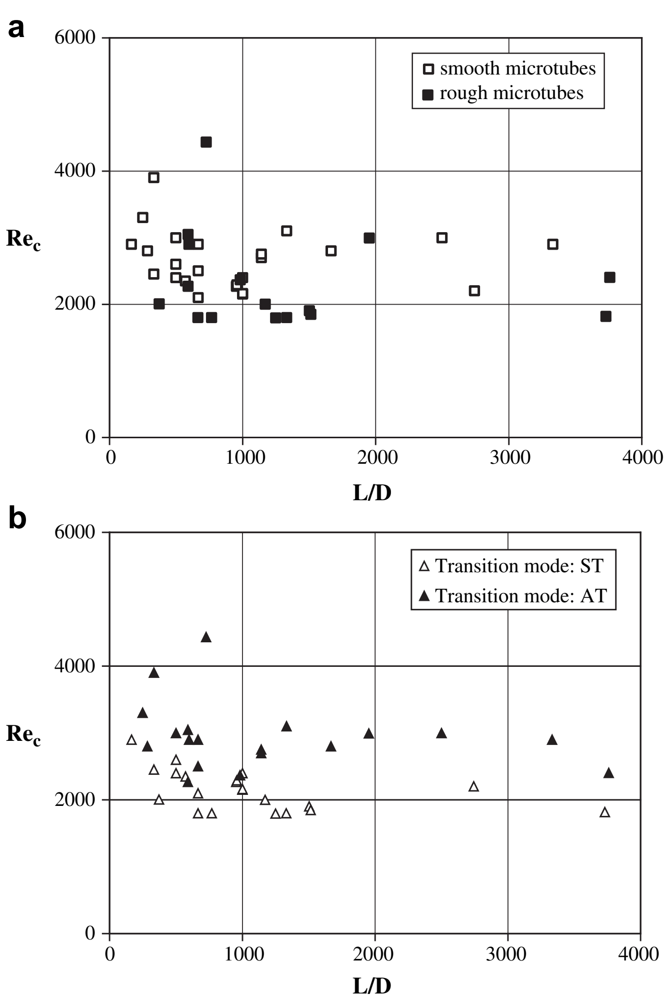

Fig. 7. Critical Reynolds numbers as a function of the *L*/*D* ratio of the microtube: role of the roughness (a) and transition modes (b).

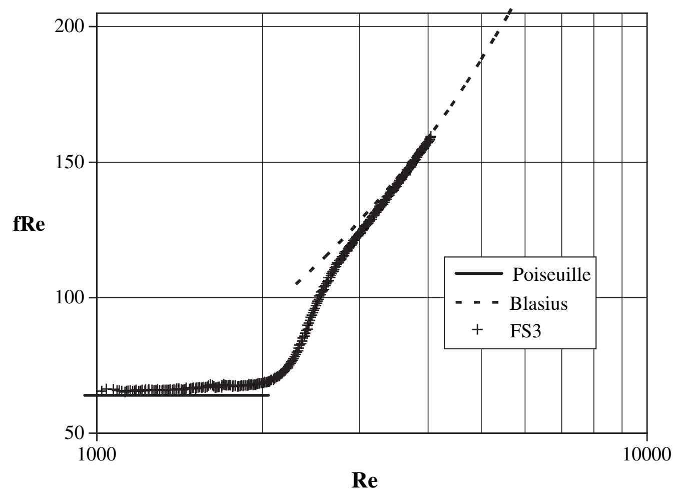

Fig. 8. Poiseuille number as a function of the Reynolds number of the fused silica FS3 microtube.

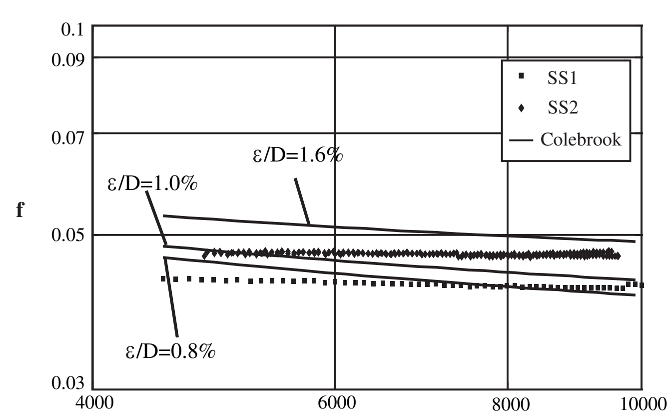

Fig. 9. Friction factor as a function of the Reynolds number in the turbulent regime.

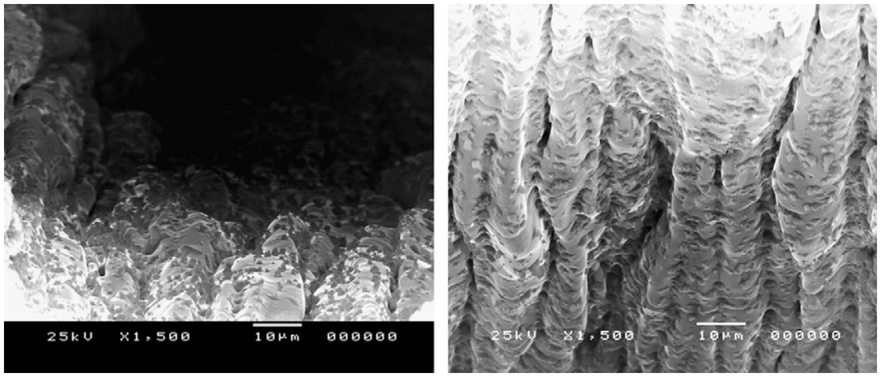

Fig. 10. Roughness features of the tested commercial stainless steel microtubes.

also the way in which these combine to yield this ratio. Delayed transition is the most common occurrence for values of the L/D ratio below 2000; yet, only six data points fall above this limit, against the forty-five falling below it, which also probably explains the apparent much larger scatter at low L/Ds. For the smooth microtubes Re_c is always larger than 2000 and never exceeds 4000.

In Fig. 7b it is evidenced that the abrupt transition (AT) tends to be experienced when the critical Reynolds number is large ($Re_c > 2300$, with one possible outlier). On the contrary, when the laminar-to-turbulent transition is characterized by low critical Reynolds numbers, it tends to be smooth and regular (ST); interestingly, this trend is shared both by smooth and rough pipes.

Although Kohl et al. [17] reported a marked increase in the value of Re_c as L/D decreases, on the basis of four data points, this trend does not seem so evident in our results, as there is a considerable scatter, most likely due to the extreme variety of inner surface, inlet and outlet of the different microtubes.

4.3. Turbulent regime

The design of the test rig enabled the observation of the fully developed turbulent regime only in the FS3, SS1 and SS2 microtubes.

In Fig. 8 the Poiseuille number of the smooth FS3 microtube is plotted as a function of the Reynolds number. It is evident that in the turbulent regime the experimental data are in very good agreement with the prediction of the Blasius law.

In Fig. 9 the friction factors obtained for the SS1 and SS2 rough microtubes for $Re > 4500$ are shown as a function of the Reynolds number. The experimental values of the friction factor in the fully turbulent regime are compared with the Colebrook–White correlation, which was validated for commercial rough tubes [33]. The agreement between the experimental results and the theoretical predictions is not very good. In particular, the experimental values of the friction factor become suddenly practically independent on the Reynolds number in the turbulent regime.

The experimental data obtained for “relative roughness” between 0.8% (SS2) and 1% (SS1) start below the corresponding Colebrook curves and evidence a weak dependence on the Reynolds number. The reason for this disagreement has been considered in detail in [34]; what is important to remark here is that the notion of “relative roughness” which is normally accepted for microchannels is not consistent with the parameter introduced by Blasius and Von Mises and employed by Colebrook in the correlation bearing his name, which is not experimental in its derivation. The ϵ used in the latter is a quantity pertinent to the surface morphology but not directly identifiable with the surface roughness as understood in engineering practice, nor should it be calculated *a posteriori* from surface measurements (which would make it most impractical). Moreover, the form of surface irregularities is also important: it has been shown that there are two types of roughness: one made up of sudden asperities of irregular shape, with short wavelength and relatively high amplitude, another more gradual, with a rather long wavelength. The former gives rise to a friction factor which is nearly independent of the Reynolds number, but very much on “relative roughness”, the latter produces a curve little dependent on the diameter and which runs higher but almost parallel to the one representing Blasius’ law. The roughness features of the commercial stainless steel tubes tested are show by the SEM images of Fig. 10, which are typical for all the microtubes tested in this experimental campaign. The distribution of the peaks is random along the perimeter with a predominant axial development of the grooves, which may also explain while transversal, artificial grooves used to simulate surface roughness (Kandlikar et al. [21]) might not be appropriate.

From the above considerations, the data plotted in Fig. 9 conform to the case of short wavelength, which also implies that for commercial microtubes the flow resistance in turbulent regime tends to become proportional to the fluid average velocity squared.

5. Conclusions

In this work experimental tests on the frictional characteristics of compressible flows in rough and smooth microtubes with diameters ranging between 29 and 508 μm are described. The main conclusions of this study can be summarized as follows:

- In the laminar regime the agreement with the conventional theory is very good both for rough (within the range of surface roughness investigated) and smooth microtubes;
- For the smaller microchannels (<100 μm) when Re is greater than 1300 the friction factor tends to deviate from the Poiseuille law; this fact can be explained by taking into account the flow acceleration due to the compressibility effect;
- There is no evidence of transition occurring earlier than predicted by the classical theory for macro-tubes; the values of the critical Reynolds number fall between 2160 and 4370 for all the tested specimens;
- Earlier-than-expected transition never occurred for the tubes tested, on the contrary it was significantly delayed, especially in rough tubes. However, an analysis of all data, including those from other experimental campaigns, show that early transition only occurs for rough tubes
- The correlation suggested by Kandlikar ([22]) does not predict the critical Reynolds number correctly for the two rough tubes tested; moreover, it sets the upper bond for transition in rough tubes at $Re = 2300$, while this phenomenon can be significantly delayed.

- Both smooth and abrupt laminar-to-turbulent transition modes have been experienced but the abrupt transition seems to become the prevailing mode when the critical Reynolds number is greater than 2300, regardless of the surface of the microtubes (i.e. either smooth or rough);
- In the turbulent regime the flow resistance is proportional to the square of the fluid average velocity, thus exhibiting little dependence on Re; the classical correlation for rough tubes (Colebrook–White) uses a "relative roughness" which is not derived from the measurement of the inner surface roughness of pipes, and this is to be kept into account when comparing microchannel data to the predictions of this correlation.

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